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STRESSED COVERINGS IN NAVAL AND AERONAUTIC CONSTRUCTION

By L. L. Kahn

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STRESSED COVERINGS IN NAVAL AND AERONAUTIC CONSTRUCTION.*

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Introduction

Aeronautic constructors have for many years been trying different kinds of stressed coverings where rigidity is secured by materials in the sheet form. Through the shape of the models and through the problems presented by their resistance, these coverings have become incorporated in the standard construction of hulls.

Generally, and particularly in naval construction, the difficulties due to the use of thin materials are summed up in rather vague expressions, such as "local weakness," "insufficient rigidity," etc. Aeronautics, however, has a very valuable means of observation, namely, the static test. The structure is loaded to the breaking point. The engineer has under his eyes a picture, almost a caricature of the distribution of the load and of the nature of the risks. The process of failure and the means which, after a failure, are successful in keeping the structure intact in the region stipulated in the specifications, show that, in structures with thin materials, the risk of buckling dominates

*"Les Bordés Travaillants en Construction Navale et Aeronautique," from a pamphlet "Association Technique Maritime et Aeronautique," May-June, 1927. This article was also published, in part, in the "Bulletin Technique du Bureau Veritas," June, 1927, pp. 119-126.

the risk of failure through simple tension. It is, however, a risk difficult to determine accurately and difficult to eliminate. This is why the new system has thus far promised more than it has given.

We propose to make a study of the difficulties and their appropriate solutions. In this elementary study, we have been compelled to follow the observations closely with the aid of calculations and simple hypotheses and to determine exactly rather than to innovate. The question of buckling has been mentioned by all who have investigated fatigue in hulls.*

The buckling has itself been studied,** but the parasitic secondary role hitherto assigned to it is too limited. The same phenomenon occurs in both naval and aeronautic construction, namely, the critical load falls in the working range, when it concerns metal plates used in the covering of hulls or very thin plates used with inadequate stiffening in the stressed covering of an airplane. We are thus led to revise the conclusions generally

*Aug. Normand: "Bulletin de l'A.T.M., 1892, "Note sur la variation du poids de charpente des navires avec les dimensions et sur la limitation qui en résulte dans la grandeur absolue."

L. Vivet: A.T.M., 1894, "Etude sur la fatigue des navires." Note, in particular, the limitation of the admissible loads for small ships.

Bertin: A.T.M., 1913 and 1914, "Esquisse d'un chapitre d'architecture navale." Note the increase in the admissible loads with the increase in the size of the ships.

Gille: A.T.M., 1915-1920, "La construction des coques métalliques, son évolution et son avenir."

**Marbec: A.T.M., 1912, "Note sur le flambement des poutres et anneaux élastiques."

Thuloup: A.T.M., 1923, "Fatigue des matériaux et sécurité des constructions."

accepted, particularly on the variation of the working stress in terms of the dimensions, a variation which plays the principal role in the limitation of the size by increasing the weight of the structure.

Notation

The millimeter is taken as the unit of length, and the kilogram as the unit of force. We will use the following symbols:

d, density;

E, Young's modulus;

R, breaking load;

M, bending moment;

I, inertia moment of a section with reference to the axis of the flexures passing through the center of gravity;

S, cross-sectional area of spar or longeron;

v, distance of matter farthest from axis of inertia;

h, thixkness of spar or longeron;

b, width of spar or longeron;

i, minimum inertia moment of covered area considered separately, i.e., the moment which enters into Euler's formula applied to the covering alone;

e, thickness of covering;

s, cross section of covering;

j^2, ratio i/s or square of radius of gyration of covering alone;

r, radius of curvature of one undulation of the covering;

l, distance between the couples;

π, ratio of circumference to diameter.

Symbols not given above are defined as they occur in the text.

I.

Advantages of stressed coverings, as applied to aeronautics. Difficulties of execution, which may destroy the theoretical advantage of weight.

On a ship the strength resides principally in the coverings of the hull and deck. The bottom of the hull and the planking, on the one hand, and the upper decks, on the other hand, constitute the flanges of a veritable girder, which transmits from bow to stern the stresses at each level. The advantage of such a structure is to cause to participate in the resistance the maximum number of longitudinal elements, whatever their role in the structure, and to impart to the sections of the girder the maximum inertia due to the utilization of the elements farthest removed from the neutral line. The ratio of weight to strength is thus reduced to the minimum.

It was with rigid airships that the transfer of the elements of resistance to the periphery made its appearance in aeronautics. Instead of securing longitudinal strength by a special girder separate from the envelope, as in nonrigid airships, this girder is composed of the framework over which the envelope is stretched. In the United States the use of duralumin less than 0.2 mm (0.008 in.) thick has even been considered

for the outside stressed covering.

On airplanes the covering was first stressed only for transmitting the external pressures at short distance between two adjacent ribs, and strength was secured by spars or lattice girders. Nevertheless, systems of stressed covering with elimination of spars has been introduced and is justified by the following considerations:

Fabric covering has the disadvantage of deteriorating in a few months when exposed to the inclemencies of the weather. On the other hand, when the wing loading of an airplane is large, the number of the ribs must be greatly increased in order to avoid excessive tension of the fabric. Hence, coverings of plywood or metal have been used. Unfortunately, while doped and varnished fabric weighs only 0.4 kg/m² (.082 lb./sq.ft.), plywood 1.5 mm (0.06 in.) thick weighs 0.9 kg/m² (.184 lb./sq.ft.) and duralumin 0.35 mm (0.014 in.) thick weighs 1.015 kg/m² (.208 lb./sq.ft.). Hence an 1800 kg (3968 lb.) airplane with a wing loading of 60 kg/m² (12.29 lb./sq.ft.) would require for wing covering alone:

24 kg (52.9 lb.) of fabric, or 1.3% of the weight;

54 " (119 ") " wood, or 3% of the weight;

60.9 kg (134.3 lb.) of duralumin, or 3.4% of the weight.

We are thus led to determine the relative strengths of the coverings in order to reduce the weight of the frame.

This is all the more desirable because, if we could localize

resisting elements in the covering, we would give the structure an inertia and consequently a rigidity greater than with the same weight of material in the form of spars or longerons.

We will consider, for example, a fuselage with four longerons covered with plywood, which can give alone, although only 1 mm (0.04 in.) and 1.5 mm (0.06 in.) thick, half of the necessary inertia. In order to be able to include it in the resistance, as also the intermediate supports which rest on the bulkheads, it suffices to arrange them in continuous, suitably combined members from one end of the fuselage to the other.

The theoretical advantage of the weight proceeds first from the elimination of the fabric and the members which support it and, secondly, from the smaller weight of the resisting members themselves. The latter cause is itself due to two causes. On the one hand, it is possible to utilize all the available height, a portion of which was occupied by the supports of the covering, the rib flanges and the nose battens. On the other hand, all the material is spread out on the surface.

Let us see, for example, what can be saved in the weight of a longeron. For this purpose, let us find how the stress in the material would vary according to the mode of construction, with equal weight and under the same bending moment. We will thus have a minimum saving in weight, in that the lightening, corresponding to the increase in the safe load, diminishes the weight, the necessary surface area and consequently the bending moments.

The safety factor of the material varies as I/v. Supposing it concerns a rectangular longeron with two like flanges and a total height or thickness h, the distance between the flanges inside the longeron being h' and the width b, then

$$\frac{I}{v} = \frac{b}{\frac{h}{2}} \frac{h^3 - h'^3}{12}$$

Put $b(h - h') = S$, cross-sectional area of longeron.

Then

$$\frac{I}{v} = S \frac{h}{6} [1 + \frac{h'}{h} + \frac{h'^2}{h^2}]$$

The values of $1 + \frac{h'}{h} + \frac{h'^2}{h^2}$ in terms of $\frac{h'}{h}$ are indicated below for different values of $\frac{h'}{h}$ between 0 (case of solid longeron) and 1 (case of longeron with infinitely thin flanges where all the material is on the surface).

Table I

$\frac{h'}{h} =$	0	0.1	0.2	0.5	0.8	0.9	0.95	0.99	1
$1 + \frac{h'}{h} + \frac{h'^2}{h^2} =$	1	1.11	1.24	1.75	2.44	2.71	2.85	2.97	3

Let us compare two longerons, one solid and, because of the covering, occupying only 0.9 of the available height, while the other occupies the whole height with an infinitely thin flange formed by the wing covering. The ratio of the I/v or of the safe loads is $\frac{3}{0.9} = 3.33$.

Inversely, the ratio of the weights, for equal factors of safety, would be 0.3 at the most. There would accordingly be a saving of at least 0.7 of the weight. Of course, this is an extreme case. When box longerons with thin flanges are used instead of solid longerons, the expression $1 + \frac{h'}{h} + \frac{h'^2}{h^2}$ approaches its maximum value. There is still a saving but of less amount.

Another advantage of this method of construction is that it does not localize the strength in separate pieces, whose failure would entail the ruin of the structure. A breaking test enables the determination of its mode of action in this case. During such a test on a metal airplane with stressed covering, we found that large blisters appeared at the factor 7, leaving no doubt but that certain regions no longer shared in the load. No break occurred, however, below the factor 12. The stresses had, in the interval, passed around the weakened areas.

It is therefore advantageous to employ this mode of construction whenever possible. It has spread very rapidly in the construction of fuselages. The fuselages are subjected, however, to much smaller couples than the wings. For example, on some cantilever monoplanes, the bending moment at their points of attachment reaches more than ten times the maximum bending moment in the fuselage, and the cross section of the fuselage is six times thicker than that of the wings.

The production of thick wings with thick junctions promoted the employment of stressed coverings for the wings. Such air-

planes are not yet made in quantity production, however, although the first attempts date back to 1915. There were encountered difficulties of principle, difficulties due to the thinness of the materials, and also serious difficulties of execution.

The principle of stressed covering consists, in short, in causing the stresses to be transmitted by surfaces instead of by linear elements. In the case of localized stresses, reinforcements are required for distributing them. This is the case, for example, when it is a question of attaching the struts or stays which, fortunately for this type of construction, are being increasingly eliminated with thick wings. But the same difficulty arises, either in demountable elements or in the case of an opening (cockpit) required for the utilization of the airplane. In both cases, it is necessary to pass from the system of stressed surface to the linear system. The smaller the airplane, the greater these difficulties appear and the greater the weight of the local reinforcements relative to the total weight. On the contrary, as the airplanes increase in size, their importance diminishes, like the local reinforcements on a ship. The difficulties due to the use of thin materials will be studied in detail when we investigate the stresses.

The structural difficulties have especially to do with the difficulty of working inside the wing in attaching the covering. Fabric can be attached without difficulty, because it can be sewed. With wood, the gluing can still be done from one side.

With metal, on the contrary, the riveting presents difficulties which we will examine more closely while reviewing the principal methods employed.

II.

Fitness of the different materials, wood, steel and light metals. Different standards of comparison. Lightness is a special advantage where there is danger of buckling. Determination of zone where buckling is to be feared. Danger of buckling of the coverings in naval construction. Limits of admissible loading. Influence on limitation of size of ships. Thinness of aeronautic materials necessitates considerable reinforcement. Comparison of different materials. Minimum stiffening required.

We will now see how the system of construction with stressed covering appears, particularly in the wings, from the viewpoint of the strength of the materials.

Table II gives the mechanical properties of the materials which have this far been used in airplane construction, i.e., wood, duralumin, and steel. It also includes magnesium and its alloys,* which have thus far been used only in engines and pro-

*We have been able to use the data on the mechanical properties of alloys and light metals according to Mr. Lecoeuvre of the "Service Technique de l'Aéronautique," who has likewise called attention to the exceptional properties of glucinum (also known as beryllium). These data were given by Mr. Lecoeuvre in a lecture before the "Société Francaise de Navigation Aérienne," not yet published.

pellers. Their use in the glider will doubtless encounter difficulties but, as we shall see, would have its advantages. Lastly, as purely indicative, we have included glucinum. This is still a laboratory product, and its price is prohibitive, but it has remarkable properties. We will disregard the questions relative to production and protection against corrosion.

Several numbers are given for each group of materials. These vary with the composition, the treatment and also according to the experimenters. Moreover, the relative strength of the joints and the mode of employment itself often modify the number to be adopted. We will doubtless never obtain very definite results in this manner. In fact, with the present dimensions of airplanes, the results prove that it is very difficult to choose. For example, in the recent contest of pursuit airplanes, the performances and weights of different airplanes present differences of the same order of magnitude, whether the material is the same or not.

Table II.

	E	R	d
Wood	1,000	3-6	0.5-0.8
Steel	22,000	40-120	7.8
Duralumin, annealed	7,800	20	2.9
" hardened	7,800	27	2.9
" treated and aged	7,800	40	2.9
Magnesium, forged*	4,500	24	1.7
Electrum, cast**	4,600	14.2	1.81
" forged	4,600	31.6	1.81
Dow metal, forged***	6,378	34	1.79
Glucinum****	32,000	40	1.8

For footnotes, see next page.

In order to study the comparative value of the materials, we will suppose them to be used on airplanes of the same type, identical in shape and dimensions, with the same total weight and the same distribution of the homologous members. Thus the test loads will be the same and identically distributed, so that the stresses and the moments in the homologous parts will be the same. Both experimentation and calculation indicate for what multiple causes the construction cannot satisfy regular requirements. None of these causes, deflection, fatigue, buckling, furnishes a standard of comparison. The choice of the best material will follow therefore from the criterion which renders superfluous the consideration of the others. This will depend on the type and especially on the dimensions.

The wing structure must in particular guard against the following risks of rupture. The air pressure on the wings is transmitted to the fuselage, which supports the principal loads, and

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*Commercial magnesium.

**Magnesium-zinc alloy with 94% Mg made by the "Chemische Fabrik Griesheim Elektron."

***Magnesium-aluminum alloy, American. On the properties and composition of light alloys, see Mr. Grard's dissertations on airplane construction. While awaiting the publication of the complete French documents, the detailed information can be found in the Journal of the Royal Aeronautical Society of London. The December, 1926, number contains a communication by W. R. D. Jones particularly devoted to magnesium and its alloys. See also the "Giesserei Zeitung" No. 9, 1925, which gives a little smaller numbers.

****Young's modulus is deduced from the formula $E=8\times10^5\,(d/a)^2$ in which a denotes the atomic weight and d the density. We do not yet have the experimental confirmation for glucinum, but the preceding formula of Peczalski applies well to all known metals, especially those with small atomic weights. See T. Peczalski, "Comptes Rendus de l'Académie des Sciences," Vol. CLXXV, 1923, p.500.

consequently produces bending stresses in the direction of the span, with maximum bending moments at their points of attachment to the fuselage. The bending moment is offset by the moment of the elastic forces relative to the axis of inertia of the attachment cross section, the fatigue being given by the well-known formula for girders

$$R = \frac{M}{\frac{I}{v}}$$

It is necessary to remember that the formula is applicable, especially to the stressed covering, only when the internal bracing prevents the flanges from shifting with reference to one another and, in general, only when the cross-sectional rigidity is assured. Simple compressive or tensile forces, which we will have to consider, appear in the bracing. This being the case, the security is assured, according to Saint Venant, when the fatigue given by this formula is admissible for the material employed.

This resistance to flexure, however, produces secondary effects, which are often very serious. In fact, and this is especially to be feared when the covering is thin, it is possible that, under the action of the load just defined, the compressed flange is in danger of buckling. In this event, the security, according to Euler, is not assured. We recall that the critical load of buckling is (See Appendix):

$$\frac{k\,\pi^2\,E\,i}{l^2}$$

If this risk is guarded against by greater thicknesses (and

we shall see to what thickness it will lead), the saving in weight, due to the employment of the stressed covering, will disappear.

Lastly, at the ends of the lattices which constitute the web of the spar, at the points of attachment of the covering, local stresses would be produced, due to defects of convergence of the lattices in the junctions, which might themselves endanger the structure. We are speaking of local flexures, on the covering, flexures which are exerted over short distances, but are very important, due to the magnitude of the stresses, of which the lattices may be the seat. They appear all the more, the less definite the neutral line of the covering at which the lattices should cross; for example, when the covering is cellular or corrugated. In this case, the secondary resistance in the junctions is not assured. The fatigue of the covering at the flexure is given by the same formula as for the girder, but in which I and v apply only to the covering.

The flexure must therefore be examined: according to Saint Venant; according to Euler; from the viewpoint of the secondary stresses at the junctions.

The transmission of the aerodynamic pressures in the direction of the wing chord, the action of the ailerons and the travel of the center of lift yield, on the other hand, to an effect of flexure following the ribs and to an effect of torsion which tends to twist the generatrices of the wings into helixes. In general, if the preceding conditions are fulfilled, the resist-

ance to flexure according to Saint Venant is here assured and it only remains to assure the resistance according to Euler by the same methods as for the longitudinal flexure. As to the resistance to torsion, the polar inertia of the sections is generally sufficient to prevent any great displacement of the center of lift, the system of stressed covering in fact rendering this inertia maximum.

We still have to examine the resistance to compression of the wing considered as a panel subjected to an external positive or negative pressure. This resistance is likewise assured in general when the preceding conditions are satisfied. Nevertheless the normal compression can play the role of a transverse stress on a girder exposed to the risk of buckling and consequently tends to diminish the critical load.

It does not suffice to remain below the limit of admissible fatigues, but it is further necessary to prevent the deformations in flight from attaining values sufficient to modify the pressure distribution or from initiating dangerous conditions. It is known that when the sections are indeformable, the deflections are given by the flexures and are proportional to $1/EI$.

In order to compare the different materials with one another, we are therefore led to classify them according to the following elements:

Safe load in simple compression or tension with equal weight;

Security against flexure $R/M/I/v$ or RI/v, since M is

by hypothesis the same for one airplane as for another;

$1/EI$, deflection due to bending;
EI/l^2, critical load of compressed flange.

1. Safe load in simple compression or tension, the parts being of the same weight and their cross sections inversely as their densities. The materials are therefore classed as R/d.

2. Fatigue at the flexure RI/v.

A.- Case of two thin flanges of like width:

$$I = be\,\frac{h^2}{2}, \qquad v = \frac{h}{2}, \qquad \frac{I}{v} = \frac{beh}{2},$$

b and h being constant and e varying as $1/d$. The materials are classified according to R/d.

B.- Case of two parts of like cross section. I varies as S^2, v as $S^{1/2}$. The parts are classified as $R/d^{3/2}$.

3. Fatigue at the flexure in the flange alone, for default of convergence in a junction, etc. The inertia of the flange, its width being constant, varies as e^3 and i/v varies as e^2. The materials are classified according to R/d^2.

4. Deflections. The standards of comparison are established as above. We find E/d for the tension; E/d for the deflection due to the flexure of the wing with stressed covering, if the width of the flange is constant; E/d^2 for the deflection due to the flexure, the sections remaining similar.

5. Buckling Ei/l^2. If the thicknesses of the flanges are

inversely proportional to their densities, we find that $E\ i$ varies as E/d^3. If, on the contrary, the width and thickness of the flanges are reduced proportionally, we find E/d^2.

Farther along we will give the classification of the materials according to the different criterions which we have just established. The materials are classified in the order of their importance and we have underlined the usual materials with their usual working stress in the calculations relative to a static breaking test.

These results are now more clearly shown in Figs. 1-2. Fig. 1 gives the densities as abscissas and the breaking strengths as ordinates, while Fig. 2 gives the modulus of elasticity. More exactly, the logarithms of these quantities were plotted. Thus the curves R/d = const., R/d^2 = const., E/d = const., etc., are reduced to straight lines whose dimensions are read on the corresponding scales. It is thus obvious that, in proportion as the degree of the denominator increases, the scale of classification approaches the scale of the densities.

The essential conclusion from these figures is the following: If we compare the materials from the viewpoint of their suitability for stressed coverings, in the form of sheet metal or boards, we find that when there appear secondary risks of buckling or of rupture for default of convergence in the junctions, the less dense materials are the best. This is because these are the ones which, for a given weight, offer the maximum local inertia.

Thus the security according to Saint Venant is classified by R/d and E/d. It assigns to the usual materials neighboring places and, excepting for very great differences in density, the classification is determined by the mechanical properties. On the contrary, the secondary risks and, more especially, the security according to Euler, which is classified according to E/d^3, gives the advantage to the light materials, headed by wood.

Table III.

Classification in R/d		$R/d^{3/2}$	
Glucinum	22.2	Wood (R=7; d=0.5)	19.35
Dow metal, forged	18.9	Glucinum	16.7
Electrum, forged	17.45	Dow metal	14.15
Steel at 120 kg/mm²	15.4	<u>Wood</u> (R=5; d=0.5)	13.8
<u>Magnesium</u>	14.1	Electrum, forged	13.15
Wood (R=7; d=0.5)	14.0	<u>Magnesium</u>	10.9
Duralumin, treated and aged	13.8	Wood (R=7; d=0.8)	9.8
		Wood (R=3; d=0.5)	8.3
Steel at 100 kg/mm²	12.8		
<u>Wood</u> (R=5; d=0.5)	10.0	Duralumin, treated and aged	7.72
<u>Duralumin, hardened</u>	9.3	Wood (R=5; d=0.8)	7.00
Wood (R=7; d=0.8)	8.75	Electrum, cast	5.94
Electrum, cast	7.85	Steel at 120 kg/mm²	5.53
Duralumin, annealed	6.9	Duralumin, hardened	5.21
Wood (R=5; d=0.8)	6.25	Steel at 100 kg/mm²	4.61
Wood (R=3; d=0.5)	6.00	Wood (R=3; d=0.8)	4.2
Steel at 40 kg/mm²	5.13	Duralumin, annealed	3.86
Wood (R=3; d=0.8)	3.75	Steel at 40 kg/mm²	1.84

Table III (Cont.)

R/d^2	
Wood (R=7; d=0.5)	28.00
Wood (R=5; d=0.5)	20.00
Glucinum	12.35
Wood (R=3; d=0.5)	12.0
Wood (R=7; d=0.8)	10.9
Dow metal, forged	10.45
Electrum	9.65
Magnesium	8.3
Wood (R=5; d=0.8)	7.85
Duralumin, treated and aged	4.75
Wood (R=3; d=0.8)	4.7
Electrum, cast	4.35
Duralumin, hardened	3.21
Duralumin, annealed	2.38
Steel at 120 kg/mm^2	1.98
Steel at 100 kg/mm^2	1.64
Steel at 40 kg/mm^2	0.66

Table IV.

E/d		E/d^2		E/d^3	
Glucinum	17790	Glucinum	9900	Wood (d=0.5)	8000
Dow metal	3540	Wood (d=0.5)	4000	Glucinum	5500
Steel	2820	Dow metal	1965	Wood (d=0.8)	1955
Duralumin	2680	Wood (d=0.8)	1570	Dow metal	1090
Magnesium	2640	Magnesium	1555	Magnesium	916
Electrum	2540	Electrum	1400	Electrum	775
Wood (d=0.5)	2000	Duralumin	926	Duralumin	319
Wood (d=0.8)	1250	Steel	362	Steel	46.5

These two viewpoints coexist in all airplanes, though in different degrees. In the usual structure with spars, or more exactly when the stresses are localized in special members, the security according to Saint Venant plays the principal role. On the contrary, when the stresses are transmitted by the surfaces, as in the case of stressed coverings, the security according to Euler, dominates. The stressed covering does not, however, exclude the local forces, no more than the systems with spars eliminate the danger of buckling. This is why we do not arrive at any definite conclusions on the basis of the estimated weight of airplanes designed.

For illustration, let us seek, in the particular case under consideration, (namely, stressed coverings of sheet metal or wood) the limit of action of the different criterions and, more precisely, the limit between the security according to Euler and the

security according to Saint Venant. This amounts to determining under what conditions the load on the flange, at the moment when the bending causes the girder to break, is less than the critical buckling load. We will then see how this condition can be artificially modified so as to eliminate the risk of buckling.

Since the flange is very thin, it may be assumed that all the matter is equally distant from the rotational axis of the cross sections when the girder bends, and that consequently its loading is uniform. The condition is then expressed by $R_s < \frac{k\pi^2 E i}{l^2}$, l being the free distance between the points of attachment of the covering; and k, a coefficient which varies from 1 to 4, when the covering is held between two points, according to the degree of fixity. Let us recall that we take

$k = 1$ for 2 free hemispherical ends;

$k = 2$ for 1 free end and 1 fixed built-in end;

$k = 4$ for 2 fixed ends.

Let us note further that R may denote not only the breaking load, but more often the load which it is not desired to exceed in the designing. If we put $j^2 = \frac{i}{s}$, in order to introduce the radius of gyration of the covering, it becomes

$$\frac{j^2}{l^2} > \frac{1}{k\pi^2}\frac{R}{E}$$

In order to fix the ideas, let us assume that the value of the degrees of fixity of the covering renders it possible to assign to k the value 2 and that the distance between the ribs or

the bulkheads is 600 mm (23.62 in.) and calculate the limiting thicknesses for the different materials employed in plates or boards of the thickness e. Then $j^2 = \frac{e^2}{12}$ and we find that $e > 467 \left(\frac{R}{E}\right)^{1/2}$ mm. For the limiting values of e, we obtain the numbers in Table V.

Table V.

Glucinum	16.5 mm	Wood at 5 kg/mm²	33.0 mm
Steel at 40 kg/mm²	19.8 "	Duralumin, treated and aged	33.4 "
Duralumin, annealed	23.6 "	Magnesium	34.2 "
Wood at 3 kg/mm²	25.6 "	Dow metal	34.2 "
Electrum, cast	26.0 "	Steel at 120 kg/mm²	34.4 "
Duralumin, hardened	27.5 "	Electrum, forged	38.8 "
Steel at 100 kg/mm²	31.5 "	Wood at 7 kg/mm²	39.0 "

These values are very similar for the usual materials.* In this list there are only two thicknesses corresponding to actual construction: 19.8 mm (0.78 in.) at 40 kg/mm² (56,894 lb./sq.in.), corresponding to the dimensions in naval construction; 25-33 mm (0.98-1.3 in.) for wood, corresponding to boat hulls or to the wings of 3-10 ton airplanes with stressed coverings.

*This is because R/E differs but little for these materials. Since this coefficient renders it possible to classify the materials according to the magnitude of the deflections for the same safety factor, the values are given in Table VI.

Table VI.

Glucinum	0.00125	Wood at 5 kg/mm²	0.00500
Steel at 40 kg/mm²	0.00180	Duralumin, hardened and aged	0.00512
Duralumin, annealed	0.00256	Magnesium	0.00535
Wood at 3 kg/mm²	0.00300	Dow metal	0.00535
Electrum, cast	0.00310	Steel at 120 kg/mm²	0.00540
Duralumin, hardened	0.00346	Electrum, forged	0.00690
Steel at 100 kg/mm²	0.00455	Wood at 7 kg/mm²	0.00700

These numbers must be regarded, moreover, as simply indicating an order of magnitude. For example, on ships the steel is stressed well below 40 kg/mm², (56,894 lb./sq.in.), which reduces the critical thickness as above defined. It is true that l exceeds 600 mm (23.62 in.), but it will be seen that it is just because of the risk of buckling that it is not used so much. When this risk is eliminated, as we shall show farther on, the working stress can be increased.

Let us solve the inverse problem. Given e, l, k, we find the critical load of buckling to be

$$R = \frac{2\ \pi^2\ E}{12}\ \frac{e^2}{l^2}$$

For steel, E = 22,000. Consequently, $R = 36,000\ \frac{e^2}{L^2}$.

Let us take for l and e the values given in the rules of the "Bureau Veritas" for steel ships: spacing of the ribs between the collision bulkhead and the caboose; thicknesses of outside

covering of the bottom and of the middle wall (1926 edition, Table II).

Table VII.

Longitudinal number	l mm	e mm	Critical load kg/mm²
170-305	555	7.0	5.7
1065-1465	605	9.0	8.0
7350-8740	705	13.0	12.2
16000-18330	765	15.5	14.8
33100-36800	840	18.5	17.5
65000-71000	930	22.0	20.0

The critical load is very small in the small scantlings, where it limits the admissible flexure, but it increases in the large structures. This is because e/l increases with the longitudinal number. In any case, it is remarkable that the progression of the admissible loads is precisely the one pointed out to the "Association Technique Maritime" by the authors mentioned, after experimental confirmation. This limitation should therefore probably be attributed to the buckling and can be calculated by Euler's formula.*

It follows from this that, in small ships, there is no advantage in using high-resistance steels for the coverings. It

*In order to justify this increase in the admissible load, it has been suggested that it was only apparent and that, in reality, the bending moment increases less rapidly than is supposed. Nevertheless, the increase in length and the relative diminution of the depth, due to the limitation of the water draft, render it difficult to assume constancy of the bending fatigues with increasing dimensions, the relative weight of the hull remaining the same.

is, in fact, E and not R which limits the admissible load. It could, on the contrary, be increased by the methods which we will study in further detail in connection with aeronautic construction. It is obviously very difficult to predict the moment when the weight of the structure will limit the increase in the dimensions. All depends, in fact, on what hypothesis is made on the variation of the working stress. If the same is retained as for small structures, it would surely be pessimistic. If, with the same material, the variation in the working stress should be extrapolated, it would be optimistic because, after reaching the thickness for which buckling is no longer to be feared, the working stress must remain constant. At this moment, however, a stronger steel can be chosen and thus the admissible stress can be increased anew.

In aircraft construction, the critical thicknesses, except for wood, are much greater than the thicknesses corresponding to the present size of our airplanes. It would not be possible, therefore, with the system of stressed covering, to use the materials at their full mechanical possibilities, because their failure by buckling would precede their failure in Saint Venant. For example, in the first static test of a small airplane of 1500 kg (3307 lb.) of 1.6 mm (0.063 in.) duralumin, the working stress did not exceed 10 kg (22 lb.) for a maximum static-test coefficient of 9. On stiffening with 1 kg (2.2 lb.) of supplementary material, it was possible to pass to the coefficient 12,

the load in the covering reaching an average of nearly 17 kg/mm^2 (24,180 lb./sq.in.). It is obvious that, even after this stiffening, we are still far below the breaking strength of duralumin.

Before passing to the measures which enable the elimination of the risk of buckling outside the limits of employ, it is interesting to see on what element the critical thickness of the covering depends and to examine the role of the coefficient $l/\sqrt{k}$. In order to make it as small as possible, we cannot think of diminishing l, that is, of multiplying the ribs because, in this way, we would lose what we would gain on the flanges, but it is necessary to make k maximum, i.e., to make it give to the covering the maximum degree of fixity at each rib. For this reason, the ribs must not be very light, as in the usual structures, but veritable bulkheads fixing, at the same time, the position and direction of the covering with reference to the attachment. They must have a good width and their flanges must be carefully connected with one another, in such manner as not to deform, and especially not to warp one another. If we could thus make $k = 4$, the preceding thicknesses would be divided by $\sqrt{2}$.

Another conclusion from this formula is that there is no advantage in varying the thickness of the covering, for example, from the fixed end to the tip of the wing. There is danger, in fact, of going below the critical thickness and thus being unable

to utilize fully the resistance according to Saint Venant. It would be better to diminish as much as possible the chord of the wing on which the covering is subjected to stress, so that, taking account of the variation in the thickness of the wing, we would not have to vary the thickness of this covering. This system consists in not trying to subject the covering to stress on a portion of the wing chord and in returning to the box girder system.

These results can be greatly improved by increasing $j^2 = \frac{i}{s}$ or, more precisely, the ratio $m = \frac{j}{e}$ which characterizes the local inertia of a covering of given thickness. For a sheet $\frac{i}{s} = j^2 = \frac{l^2}{12}$,

$$m = \frac{j}{e} = \frac{l}{\sqrt{12}} = 0.289.$$

The condition $R_s > \frac{k\pi^2 E i}{l^2}$, on taking account of $i = m^2 e^2 s$, is then written

$$m > \frac{l}{\sqrt{k\pi^2}} \frac{l}{e} \sqrt{\frac{R}{E}}$$

Before considering the processes employed for obtaining this local inertia characterized by m, let us find the values required for m in the order of magnitude of our airplanes and according to the materials used. We will put $m' = \frac{m}{0.289}$, so that $m' = 1$ for a metal plate or a board. The coefficient m' will characterize the ratio in which a material must be stiffened, in order to remove the danger of buckling; that is, the degree of

transformation to which the material must be subjected. We will call m' the coefficient of stiffening.

Let us first assume, as before, that the airplanes are of the same weight with the same distribution and, in particular, that the coverings are of the same weight. Then e varies as $\frac{1}{d}$ and m' as $d\sqrt{\frac{R}{E}}$.

If, on the contrary, the covering is subjected to the same stresses in Saint Venant, e varies as $\frac{1}{R}$ and m' as $R\sqrt{\frac{R}{E}}$, but the latter coefficient of comparison assumes that the same stresses have been retained while, for example, the weight of the covering has been replaced by a useful load producing the same effects on the structure. In practice m depends simultaneously on $R\sqrt{\frac{R}{E}}$ and on $\frac{d}{R}$. When $\frac{d}{R}$ is diminished, the weight and the surface areas can be diminished, but further stiffening is required to prevent buckling.. The result is obtained by direct calculation. Table VIII gives the classification of the materials according to the criterions $d\sqrt{\frac{R}{E}}$ and $R\sqrt{\frac{R}{E}}$, which are proportional to the coefficient of stiffening.

Table VIII.

$d\sqrt{\frac{R}{E}}$		$R\sqrt{\frac{R}{E}}$	
Wood at 3 kg/mm² (d = 0.5)	0.0274	Wood at 3 kg/mm²	0.164
" " 5 " (d = 0.5)	0.0353	" " 5 "	0.353
Wood at 7 kg/mm² (d = 0.5)	0.0409	Wood at 7 kg/mm²	0.164
" " 3 " (d = 0.8)	0.0438	Electrum, cast	0.79
" " 5 " (d = 0.8)	0.0565	Duralumin, annealed	1.01
Glucinum	0.0646	Glucinum	1.415
Wood at 7 kg/mm² (d = 0.8)	0.0670	Duralumin, hardened	1.535
Dow metal, forged	0.0955	Steel at 40 kg/mm²	1.71
Electrum, cast	0.1005	Magnesium	1.75
Magnesium	0.124	Dow metal	1.83
Duralumin, annealed	0.147	Electrum, forged	2.62
Electrum, forged	0.150	Duralumin, hardened and aged	2.86
Duralumin, hardened	0.170	Steel at 100 kg/mm²	6.75
Duralumin, hardened and aged	0.208	Steel at 120 kg/mm²	8.90
Steel at 40 kg/mm²	0.333		
" " 100 "	0.526		
Steel at 120 kg/mm²	0.577		

These results are summarized in Fig. 3, which illustrates the advantage of light materials. An examination of Table VIII and Fig. 3 shows that m increases considerably in passing from wood to the light metals and from the latter to steel. With equality of weight, steel at 100 kg/mm² (142,235 lb./sq.in.) must

be stiffened 16 times as much as wood, hardened duralumin 5 times as much, magnesium 3.5 times as much. With equality of safety stress, steel at 100 kg/mm² must be stiffened 19 times as much as wood, hardened duralumin 4.5 times as much, and magnesium 5 times as much.

Let us see to what value of m' we would be led, in the order of magnitude of existing airplanes, in order to concrete these results. For this purpose, let us begin with a 2 mm (0.08 in.) covering of duralumin at 27 kg/mm² (38,400 lb./sq.in.). Let us assume, as before, that $k = 2$ and $l = 600$ mm (23.62 in.) and determine the corresponding thicknesses of the other materials and the value of m' in all these cases, necessary to eliminate the danger of buckling. We will assume that the thickness of the covering is to be determined either by equality of weight or by equality of safety factors. In the latter case we will be able to determine the relative weights of the covering. The inversions would give the ratio of the factors of safety with equality of weight. All this is summed up in Tables V and VIII.

Table IX indicates the values to be assigned to the coefficients m or m' in order to be able to pass from the considerable and obsolete thicknesses indicated in Table V to the thicknesses which would be used in stressed coverings on present-day airplanes. These values lie between $m = 1.41$ (a value very easy to obtain) for woods used under a small load, and 77 for steel at 120 kg/mm² (170,682 lb./sq.in.). They would be dimin-

ished or increased according as one would be led by the calculation of Saint Venant to employ thicknesses greater or smaller than 2 mm (0.08 in.) for duralumin. They are inversely proportional to the thickness and directly proportional to the distance between bulkheads and to the 3/2 power of the assumed load. It should be noted that m does not affect the weight of the structure and characterizes only a form of employment of the materials.* Lacking an adequate value of m, supplementary weights would have to be used. Generally, however, m will lead to high conversion costs or to difficulties in mounting, i.e., to an increase in the cost of construction. The use of standard section metal or plates might, however, reduce construction costs to values which would not be prohibitive.

Section III will show how there have been given or how it has been proposed to give to m values of the order of magnitude of those indicated in the preceding table.

*If Peczalski's law were exact, m would be proportional to the product of the assumed load multiplied by the atomic weight.

Table IX.

Comparison of thickness, weight and stiffening, according to the materials used.

	Equality of weight $m=23.3d\sqrt{\frac{R}{E}}$.	$m'=80.7d\sqrt{\frac{R}{E}}$.	
	m 1	m' 2	e 3 mm
Wood at 7 kg/mm² (d = 0.5)	0.955	3.3	11.6
" " 7 " (d = 0.8)	1.56	5.4	7.25
" " 5 " (d = 0.5)	0.322	2.84	11.6
Wood at 5 kg/mm² (d = 0.8)	1.317	4.55	7.25
" " 3 " (d = 0.5)	0.638	2.21	11.6
" " 3 " (d = 0.8)	1.02	3.53	7.25
Steel at 40 kg/mm²	5.75	19.9	0.75
" " 100 "	12.5	43.2	0.75
Steel at 120 kg/mm²	13.42	46.5	0.75
Duralumin, annealed at 20 kg/mm²	3.42	11.8	2.00
Duralumin, hardened at 27 kg/mm²	3.96	13.7	2.00
Duralumin, aged at 40 kg/mm²	4.84	16.7	2.00
Magnesium at 24 kg/mm²	2.88	10.0	3.4
Electrum, cast	2.32	8.05	3.2
" forged	3.5	12.1	3.2
Dow metal at 34 kg/mm²	2.22	7.7	3.24
Glucinum at 40 "	1.51	5.22	3.22

Table IX (Cont.)

Comparison of thickness, weight and stiffening, according to the materials used.

	Equality of safe stresses			
	$m = 2.5R\frac{R}{E}$.		$m' = 8.65R\frac{R}{E}$.	
	m 4	m' 5	e 6	Relative weight 7
			mm	
Wood at 7 kg/mm² (d = 0.5)	1.465	5.07	7.7	0.66
" " 7 " (d = 0.8)	1.465	5.07	7.7	1.06
" " 5 " (d = 0.5)	0.883	3.05	10.8	0.93
Wood at 5 kg/mm² (d = 0.8)	0.883	1.412	10.8	1.49
" " 3 " (d = 0.5)	0.41	1.412	18	1.55
" " 3 " (d = 0.8)	0.41	1.412	18	2.48
Steel at 40 kg/mm²	4.26	14.7	1.35	1.81
" " 100 "	16.8	58	0.54	0.727
Steel at 120 kg/mm²	22.2	77	0.45	0.605
Duralumin, annealed at 20 kg/mm²	2.52	8.7	2.7	1.35
Duralumin, hardened at 27 kg/mm²	3.96	13.7	2.0	1.00
Duralumin, aged at 40 kg/mm²	7.15	24.7	1.35	0.674
Magnesium at 24 kg/mm²	4.47	15.5	2.25	0.66
Electrum, cast	1.93	6.7	3.8	1.18
" forged	6.5	22.5	1.71	0.532
Dow metal at 34 kg/mm²	4.57	15.8	1.59	0.49
Glucinum at 40 "	3.54	12.25	1.35	0.422

III

Different ways of imparting the necessary rigidity to the covering. Multiple thicknesses, stiffeners, corrugations. Proposed solutions. Practical difficulties.

The proposed methods of construction might be classified systematically, either from the viewpoint of rigidity or facility of fastening. The ingenuity of inventors and manufacturers is being untiringly devoted to the latter problem and we will not dwell on it here. We will consider all the possible methods:

Mounting by separate elements;

Openings which enable the passage of the hands and head and which are closed at the last moment;

Tubular rivets with eyes through which the rivets can be closed from the outside by means of a special tool;

Systems designed to arrange the joints in such a way that the covering can be riveted entirely from the outside, without including, for exceptionally difficult riveting, the use of sets of mirrors or pliers of complicated design and small-bodied apprentices who can crawl inside the wings.*

The methods used for imparting a high value to m can be classified as follows:

*This system requires a projection on the wing which is utilized only on coverings partially stressed, the joints being parallel to the ribs, Dornier method, Wibault fold.

1. Covering in multiple thickness;
2. Using stiffeners;
3. Corrugating the surface;
4. Combinations of these methods.

The first method consists in employing, instead of a single thickness e, for which $j^2 = \frac{e^2}{12}$ and $m' = 1$, for example, two plates of thickness $e/2$ separated by a distance f, for which

$$j^2 = \frac{e^2}{12}\left(1 + 3\,\frac{f}{e} + 3\,\frac{f^2}{e^2}\right);$$

$$m' = \sqrt{1 + 3\,\frac{f}{e} + \frac{3f^2}{e^2}},$$

or $\frac{f}{e}\sqrt{3}$, if f is large in comparison with e.

For example, the obtention of the values of m' indicated in Table IX, column 6, would require:*

Wood at 5 kg/mm² (7112 lb./sq.in.)	- for	f = 19.0 mm (0.75 in.);
Steel at 100 kg/mm² (142235 lb./sq.in.)	- for	f = 19.4 mm (0.76 in.);
Duralumin at 27 kg/mm² (38400 lb./sq.in.)	- for	f = 15.8 mm (0.62 in.);
Magnesium	- for	f = 19.8 mm (0.78 in.).

The components of the double covering are therefore very thin, much thinner than those employed in naval construction. If the two coverings are separated, the inertia of the single

*This amounts to giving f practically double the radius of gyration of the boards or plates whose thickness is given in Table V.

covering would be theoretically increased, but the total inertia of the girder would be decreased. Moreover, for the two layers to be included in the inertia of the covering, it is necessary for the connections between them to be adequate, so they can be considered as constituting a single covering.*

Obviously there is a difference between such a double covering and those employed in naval construction. In ships it is made by causing to share in the resistance the plates near the end plates, made continuous for this purpose and which would otherwise constitute a dead weight from the viewpoint of resistance, but the mutual aid of two adjacent plates in preventing their individual buckling between beams is due largely to section irons placed at the ends of the diagonals and floor boards or to the connecting bulkheads. Nevertheless, the total inertia of the cellular region helps to prevent the buckling of the whole between the supports constituted by the large transverse frames or bulkheads. In aeronautical construction it is difficult to work in the small space enclosed in the double shell, so that the use of special section irons or wooden blocks shaped in advance has been considered.

The use of stiffeners makes it possible to employ a single covering. The inertia of the covering is reinforced locally. If the connection between the stiffener and the covering is adequate,

*Of course, the connections and the longitudinal partitions can be included in the section of covering, so that the thickness e, as given in Table IX, can be somewhat reduced.

so that the local rotation of the plating entails that of the stiffener, we can take, as the inertia of the covering, the inertia of the figure about an axis passing through the center of gravity of the whole section of the plate, including the stiffener. The inertia of the plate is thus increased by adding to it the inertia of the stiffener and by replacing the neutral axis through the median line of the plate with a more remote neutral axis.

In order to produce this condition, the stiffeners must be very near one another, which entails great difficulties of execution. We are then led to use large stiffeners, at too long intervals, so that we cannot introduce the total energy of the covering into Euler's formula. The neutral axis moves away from the plate on the right of each stiffener and returns to the plating at the middle of the intervals. Experience proves that, at the time of the static test, there is a blistering between the stiffeners, so that the latter are the only stressed elements and must be reinforced anew, when the plating, reduced to the role of covering, is too heavy for this purpose alone. The system of stressed covering is therefore converted into a system with multiple longerons with the aggravating condition that the flanges are not adequately joined and there is no economy in weight nor gain in security. Nevertheless, a stiffener can improve a plate which is lacking in rigidity. It not only helps with its own rigidity, but it stiffens quite a zone of the covering in its vicinity.

For example, let us calculate a stiffener composed of bands normal to the plating, equidistant and of the same thickness e_1, as the plating. Let us assume their width to equal their equidistance $p_1\ e_1$. Then the axis of inertia is at a distance $\frac{p_1}{4}\ e_1$ from the plating and the inertia of a band of width $p_1\ e_1$ becomes

$$e_1\left(\frac{p_1}{12} + \frac{5p_1^3}{24}\right),$$

The section is $2p_1\ e_1^2$ and the radius of gyration is

$$e_1\sqrt{\frac{1}{12} + \frac{5p_1^2}{24}}$$

or practically

$$j = 0.46\ p_1\ e_1.$$

If it is assumed that we have to do with duralumin at 27 kg/mm² (38400 lb./sq.in.) held every 600 mm (23.62 in.) with $k = 2$, the value of j can be deduced directly from Table V.

$$j = \frac{E}{\sqrt{12}} = \frac{27.5}{\sqrt{12}} = 8 \text{ mm}$$

If it is assumed that the same section be employed as in the simple covering whose thicknesses are given in Table IX, then e will be 1 mm (0.04 in.) instead of 2 mm (0.08 in.) and we will have $p_1 = 17.5$ mm (0.69 in.). Thus an adequate stiffening can be obtained by substituting, for a single plate 2 mm thick, a 1 mm plate stiffened every 17.5 mm by a band 1 mm thick and 17.5 mm wide. In the same manner we find for the interval and width of the stiffeners:

For wood at	5 kg/mm^2	(7112 lb./sq.in.)	21.2 mm (0.835 in.);
" steel "	100 "	(142235 ")	20.2 " (0.795 ");
" magnesium			21.9 " (0.862 ").

In the same manner we could calculate the distribution of the other stiffeners, such as the standard sections, bulb angles, etc., used in naval construction. The principal difficulty (lessened in naval construction by hot riveting and welding) is to secure a close union of the plates and stiffeners.

Corrugations likewise render it possible to increase the inertia of the covering with only a single thickness. Moreover, they are easy to make. Let us assume, for example, that they are semicircular with a radius of r, if we disregard e^2/r^2 before unity, the inertia of such a plate is $\pi e r^3$ and $j = \frac{r}{\sqrt{2}}$ instead of $\frac{e}{\sqrt{12}}$ for a flat plate.

$$m' = \sqrt{6 \frac{r}{e}}.$$

Experience shows that the result of this calculation is optimistic, due to the fact that we have considered a series of complete corrugations, like so many closed tubes. As in the case of the stiffeners, the line of inertia of the covering is not a straight line, but an undulating curve, which approaches the covering more or less closely.

Moreover, the corrugations, which should be employed in the direction of the span, have the serious disadvantage of imparting a sinuous path to the air flow around the wing and of thus

diminishing the fineness of the profile. Hence they are employed externally only in the direction of the ribs. In the latter case, the covering can no longer be regarded as a veritable stressed covering. These corrugations are therefore associated only with an internal structure of spars or girders. A method derived from the latter consists in utilizing corrugations as local stiffeners.

Of course the various processes just enumerated can be used to stiffen the plates in the direction of the span or in the direction of the chord; also crossed corrugations and stiffeners or cellular systems with rectangular meshes.

Figs. 4-16 illustrate the methods employed by various constructors, as taken from patents issued in France.

The distances are about five times as great in Isherwood's system (Fig. 4) as in the classic system and the danger of buckling had to be combated by special stiffeners.

Stressed-covering construction appears to have been first introduced into aeronautics by the Nieuport fuselage, but it was in Germany, and doubtless because of the experience acquired in the construction of Zeppelins, that the rules of construction applicable to stressed wing coverings appear to have been enunciated for the first time in nearly complete form (1915-1918). We will mention the methods of Junkers, Zeppelin, and Dornier. Since the war, the number of researches and inventions in this connection has greatly increased in France (S.I.M.B., Hubert, Dewoitine,

De Boysson, Wibault, Kahn). We will give a few characteristic examples.

Conclusions

The chief conclusion from the viewpoint of the employment of stressed coverings in aeronautics is that the structural problem is to use the minimum amount of matter to give the inertia and, more accurately, the radius of gyration necessitated by the danger of buckling. The smaller the airplanes and the denser the materials composing them, the greater the risk of buckling. Instead of overcoming it by the difficult and expensive means which we have passed in review, weight is lost instead of gained on the classic types of construction. We have not dwelt on all the difficulties. The methods we have described are often silent on the question of attachments and openings and generally on the union with the stressed linear elements. In this connection, it is possible to lose all the theoretical saving in weight.

The results are not yet brilliant. In spite of the role at first played by Junkers and Dornier, the covering is not stressed longitudinally in either the Junkers commercial airplane or the Dornier "Wal." In France there have been several very interesting tests and many investigations, but the only practical airplane constructed and tested in flight is no lighter than airplanes of the same class of standard design.

Nevertheless, the question is an important one, because the smallest saving in weight is valuable in practice and the trans-

mission of stresses by surfaces instead of beams increases the safety, especially from the military viewpoint.* It is because, thus far, the economic conditions of aviation have necessitated keeping the construction costs near those of the previous methods of construction, that inadequate values of the coefficient of stiffening have been attained and that not all the economy of weight of which the system is capable, has been found. There is need of pursuing the problem further and improving the results.

These difficulties grow less, as the airplanes grow larger. The structural problem will join that of naval construction, where the system of stressed coverings is the conventional system. The transition will be easier, if demountability can be dispensed with, and if wood construction is retained, because the coefficient of stiffening is smaller and consequently the transformation of the crude material less difficult. Lastly, when we can use "extra light" alloys in airplanes, the employment of metal coverings will be facilitated.

In ships the secondary risks appear to play the same role as in airplanes, and it seems that buckling must participate directly in the calculation of the hulls. The comparisons which we have been able to make, show that we have the means for calculating the effects. Fortunately the thicknesses required are such that they can be easily improved without necessitating, as

*De Fleury, "Les métaux légers au dernier Salon de l'Aéronautique" in "Technique Moderne," January and February, 1927.
Hauser, "Influence économique de la légèreté dans la construction des navires" in "Bulletin de l'Association Technique Maritime," 1890.

in aviation, the veritable elaboration of a special material.

Appendix

Note on the Critical Load of Buckling.

Different Formulas Proposed for Determining It.

In the present treatise we have always used Euler's formula. Very many experiments, however, have shown that it would be better, in certain cases, to use different formulas, either Rankine's or Strand's (See article by Soulages in "Revue Generale de l'Aeronautique" No. 5, p. 124).

For the critical load with unity of section, the three formulas give:

Euler: $$\frac{k\pi^2 E}{\left(\frac{l}{j}\right)^2} = \frac{k_1}{\left(\frac{l}{j}\right)^2};$$

Rankine: $$\frac{k^2}{1 + k_3\left(\frac{l}{j}\right)^2};$$

Strand: $$k_4 e^{-k_5\left(\frac{l}{j}\right)^2},$$

which can be written

$$\frac{k_6}{1 + k_7\left(\frac{l}{j}\right)^2 + \dots}.$$

We did not think best to resort to the formulas of Rankine

and Strand for the following reasons:

1. When it is a question of comparing materials with one another, whatever formula is used, consideration of buckling always involves the comparison of i/l^2 for the different materials because, in all the formulas, it is always in terms of i/l^2 that the breaking load is determined. Consequently, there would be nothing to change in our conclusions, if the formula of Euler were replaced by either of the other formulas.

2. When it is a question of determining the i/l^2 of the covering which gives the buckling, we have kept within the zone where the different writers agree that Euler's formula applies.

3. Moreover, Euler's formula corresponds to a well-defined object and appears to be beyond criticism, when it is isolated. It assumes that the buckling girder fails through the conventional process of flexure. This process presupposes essentially the indeformability of the section whose inertia is introduced into Euler's formula, i.e., the inertia of the compressed flange, which is the case under consideration.

Euler's formula is therefore applicable except in the two following cases:

a) When compression plays a more important role in the destruction than flexure, as in the case of a very short girder. If the girder is of zero length, we must find the load in pure compression, which is exactly measured on taking a very short

girder. Rankine's formula is therefore approximately applicable. It has been found to give the same results as Euler's formula when i/l^2 is very small and the breaking load when i/l^2 is very large.

b) When the cross section does not remain indeformable after the buckling begins. It must not be forgotten that the inertia of the section is introduced into the conventional calculation of the flexure, because a rotation of two adjacent sections about a neutral axis is assumed. If, therefore, there is a local buckling and not a buckling of the whole between the supports, it is at least not possible to consider the buckling as a conventional flexure, but it is an artifice for introducing into the calculation the inertia of only a portion of the section. Obviously we would thus obtain a critical load which could be incomparably smaller. This is what happened in the small tubes investigated by Soulages, which buckled under compression, but we must take for r the radius of the tube or the radius of gyration of the thin wall, or an intermediate term. When this mode of failure is possible, the load is still further reduced than in the case of buckling.

In order to avoid this, it was proposed to stiffen the walls of the tube by corrugations, or the corrugations themselves by still smaller corrugations, particularly at a long distance from the neutral fibers. Protection is thus afforded against local

buckling (Junkers patent, Fig. 6). This tendency to local buckling appears to depend on the treatment of the material. It seems therefore that we must not be satisfied with E, but must determine a particular coefficient, which is done in Strand's formula where k_4 represents the breaking load by compression of a very short tube, a load varying according to the specimen. It is then harmonized analytically with Euler's formula, when i/l^2 is very small.

There still remains in Euler's formula an indefiniteness regarding the value to be assigned to k. In practice there is no clear distinction between the attachments of the extremities. The flexibility of these attachments varies from zero to infinity, when k varies from 1 to 4. It would be desirable to undertake precise experimentation (with at least the precision of the "comparer" and of the Poggendorf mirror) on the value of k and the shape assumed after buckling, when the usual methods of attaching the coverings to the beams are employed. Such experimentation would supplement the well-known experiments of Raclot.

Influence of very Stiff Elements.- Elastic Buckling

When elements not exposed to buckling coexist with thin elements, the buckling of the latter does not entail their immediate failure. It seems that they transmit their loads to stiffer elements. This phenomenon may save a structure, when it occurs in the vicinity of the imposed limits and is explained as follows:

Let us assume, for simplicity, a model consisting of an element F.I. with small inertia, whose extremities are joined to those of an element G.I. of great inertia not exposed to buckling. Let us assume these two elements to be of equal cross section. So long as F.I. does not buckle, the compressive load is equally distributed between F.I. and G.I. Let us see how it is distributed when F.I. buckles. The distribution is controlled by the condition that the ends of G.I. and F.I. are joined and that therefore, at each instant, the chord of F.I. remains equal to the length of G.I. shortened normally according to Hooke's law. If E is the modulus of elasticity, G.I. would be shortened by $1/E$ of its length for every load increment of 1 kg/mm².

The element F.I., assumed, for example, to be held between free round ends, takes the shape of the linear element of Bernoulli. The load R and the shortening A are given by the formulas

$$R = \frac{4E}{l^2} j^2 H_1, \qquad A = 2 (1 - H_2).$$

(According to Bouasse, "Resistance des matériaux" and "Mathématiques générales.")

H_1 and H_2 depend on elliptic integrals and simultaneously take the following values:

H_1	H_2	A
$\frac{\pi^2}{4} = 2.4673$	1	0
2.4676	0.99987	0.00026
2.4691	0.99935	0.00130
2.4708	0.99873	0.00254
2.4734	0.99759	0.00482
2.4768	0.99628	0.00744

For a small overload on the critical value, the shortening is practically proportional to the overload and, on designating by E' the ratio of the overload to the elongation, we have

$$E' = \frac{4E}{l^2} j^2 \frac{H_1 - 2.4673}{2(1 - H_2)},$$

or practically

$$E' = 4.62 \, E \frac{j^2}{l^2}.$$

Let us assume F.I. to be a steel plate 10 mm (0.394 in.) thick and 600 mm (23.62 in.) long. Then

$$j^2 = \frac{e^2}{12},$$

$$E' = 1.067 \times 10^{-4} E, \qquad \text{or practically} \qquad E' = \frac{E}{10000}.$$

Consequently, when the load is increased, F.I. is stressed 10,000 times less than G.I., or 0.1g for 1 kg. The ratio varies, moreover, as e^2. It reaches 1,000,000, if $e = 1$ mm. This means that, after the element F.I. buckles, its load does not

increase beyond the critical load. The stiffeners alone support the overloads. The plate does not fail and it can return to its straight condition as soon as the compression is diminished.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

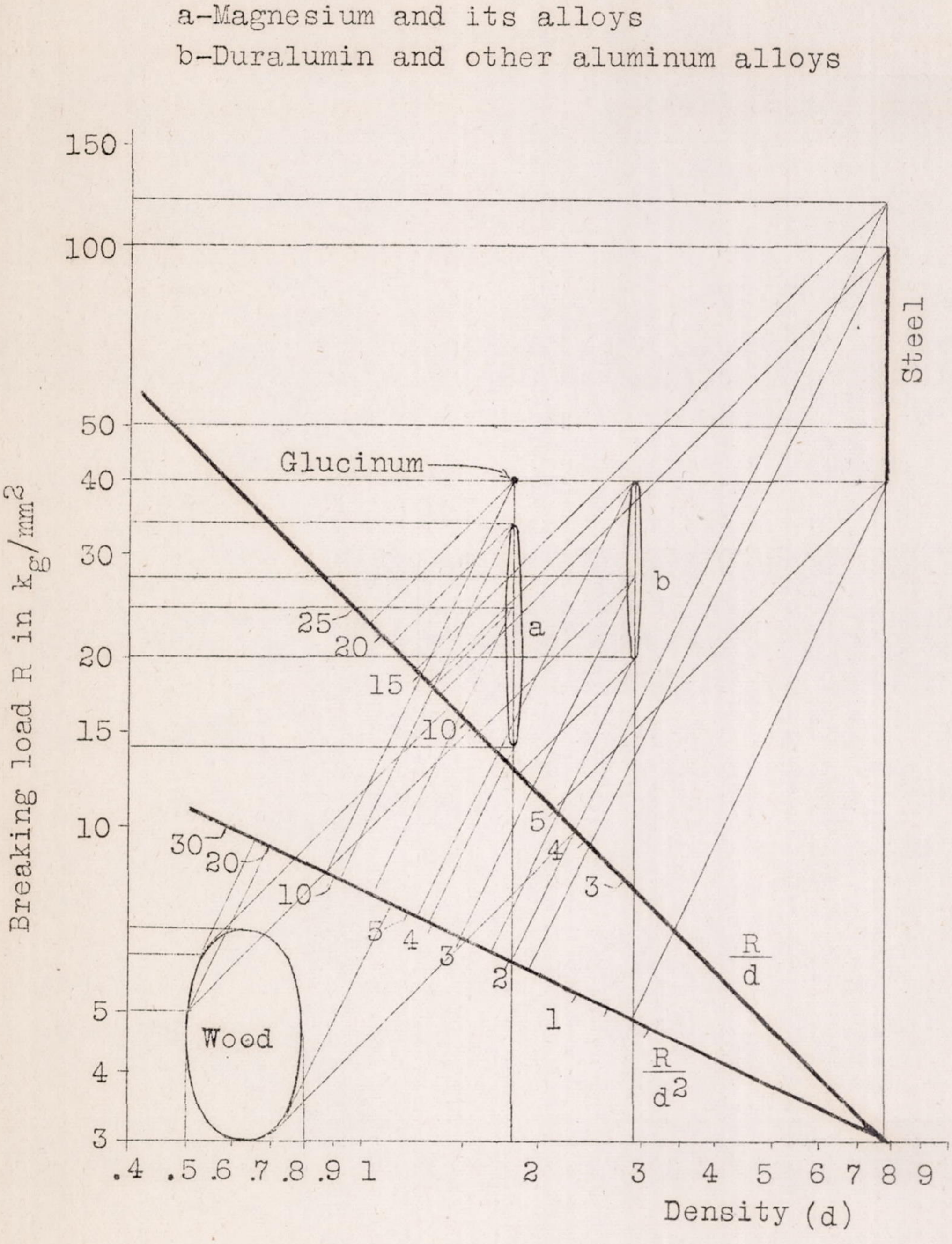

a-Magnesium and its alloys
b-Duralumin and other aluminum alloys
150
100
50
40
30
20
15
10
5
4
3
Breaking load R in k_g/mm^2
Steel
Glucinum
a
b
25
20
15
10
5
4
3
30
20
10
5
4
3
2
1
Wood
$\frac{R}{d}$
$\frac{R}{d^2}$
.4 .5 .6 .7 .8 .9 1 2 3 4 5 6 7 8 9
Density (d)

Fig.1

a-Duralumin and other Al.alloys.

b-Magnesium and its alloys.

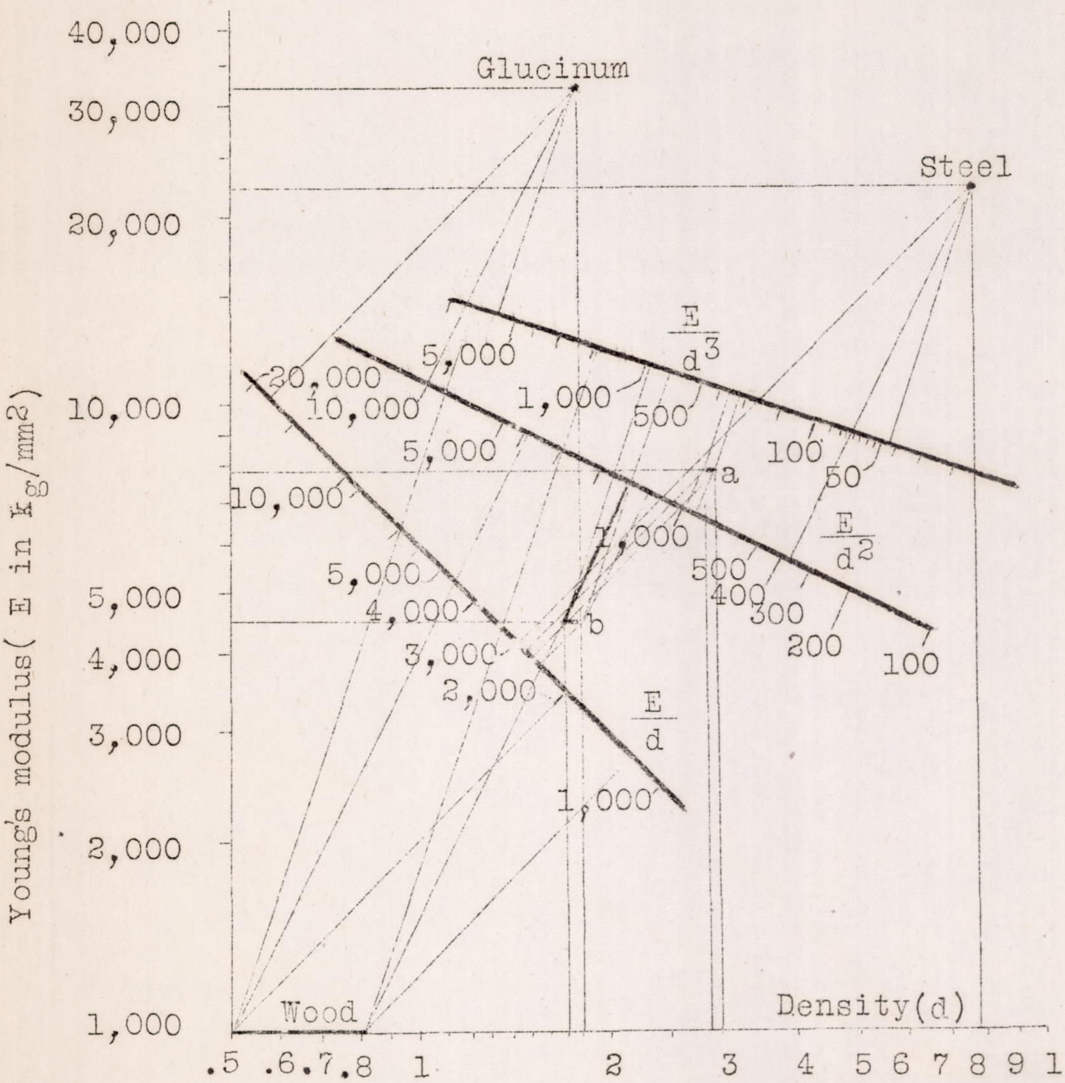
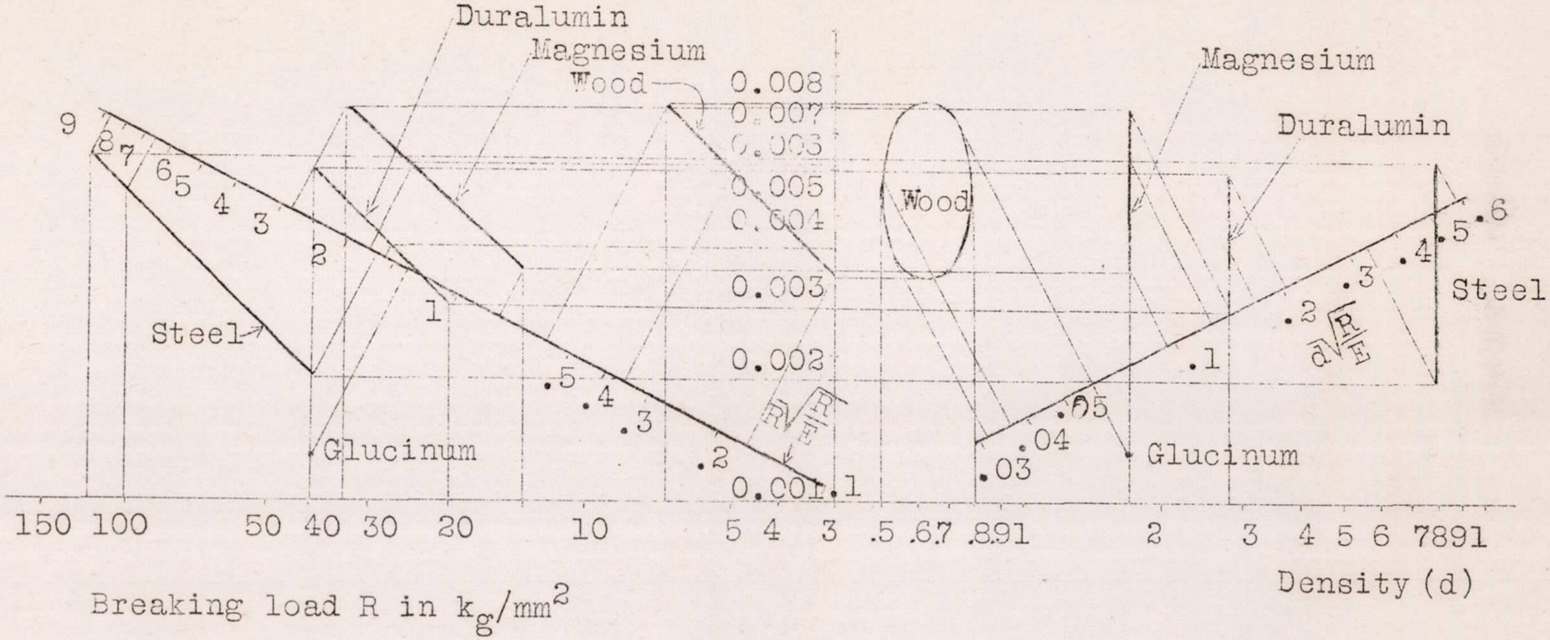


Fig.2



Duralumin
Magnesium
Wood
Steel
Glucinum
Wood
Magnesium
Duralumin
Steel
Glucinum
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
150 100 50 40 30 20 10 5 4 3 .5 .6 .7 .8 .9 1 2 3 4 5 6 7 8 9 1
Breaking load R in kg/mm2
Density (d)

Fig.3

Fig.3

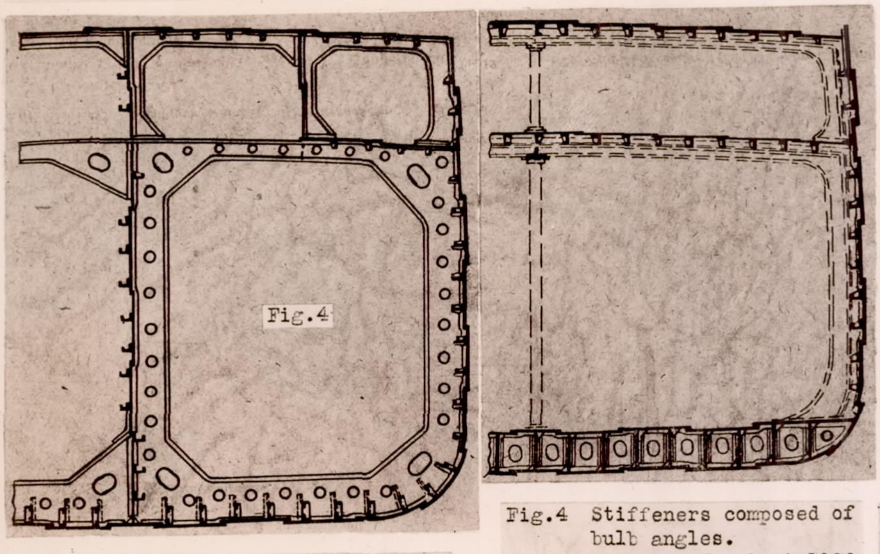


Fig.4 Stiffeners composed of bulb angles. I.W.Isherwood patent,1908

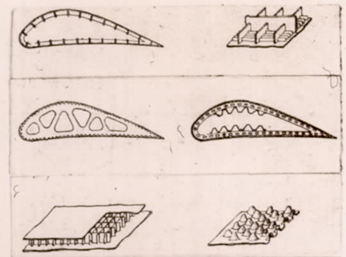


Fig.5 Cellular system,with stiffeners and with corrugations. H.Junkers patent.

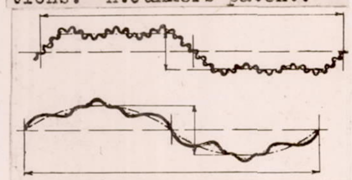


Fig.6 Superposition of corrugations of different orders. (See appendix) H.Junkers pat.

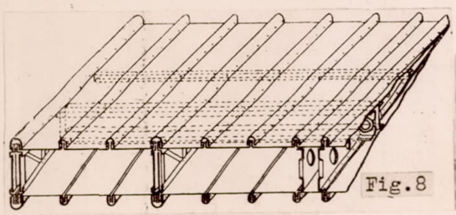


Fig.8

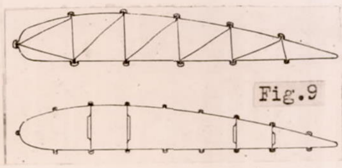


Fig.9

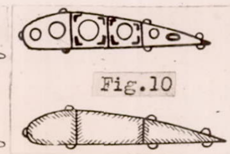
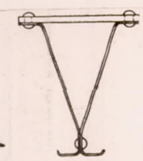


Fig.10



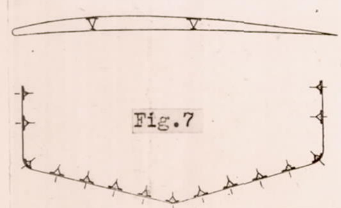


Fig.7

Figs.7,8,9,10 System of external seams. A similar method is employed on the Wibault airplanes. Dornier pat.

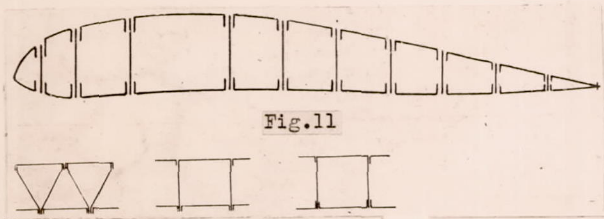


Fig.11

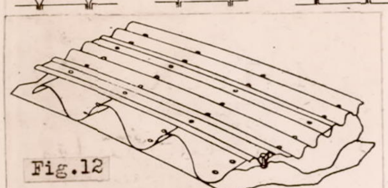


Fig.12

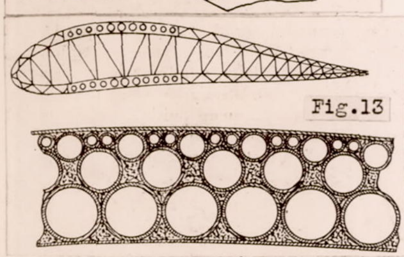


Fig.13

Figs.11,12,13 System of internal joints,with different types of cellular covering having great local inertia. Openings in the panels for work and inspection. S.I.M.B. patents, Belgium.

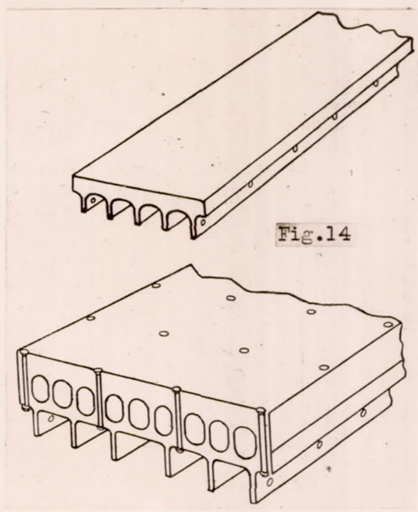


Fig.14

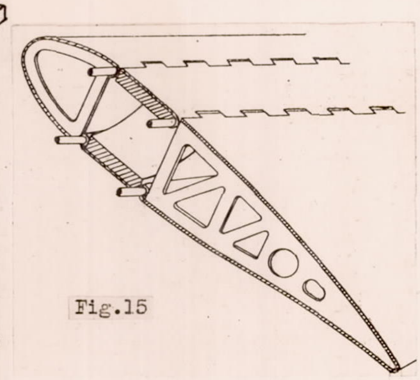
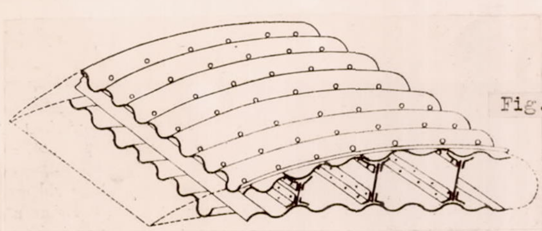


Fig.15

Figs.14,15 Type of cellular covering. Dewoitine patents.



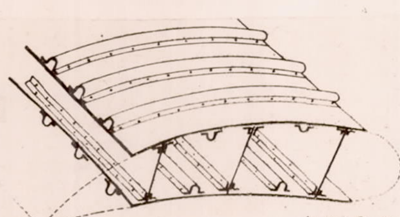


Fig.16

Fig.16 Stiffeners or crossed corrugations. A.DeBoysson patent.

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